Sofia Leal Cavazos
Design Project

April 30, 2024
CE 331 – Reinforced Concrete Design
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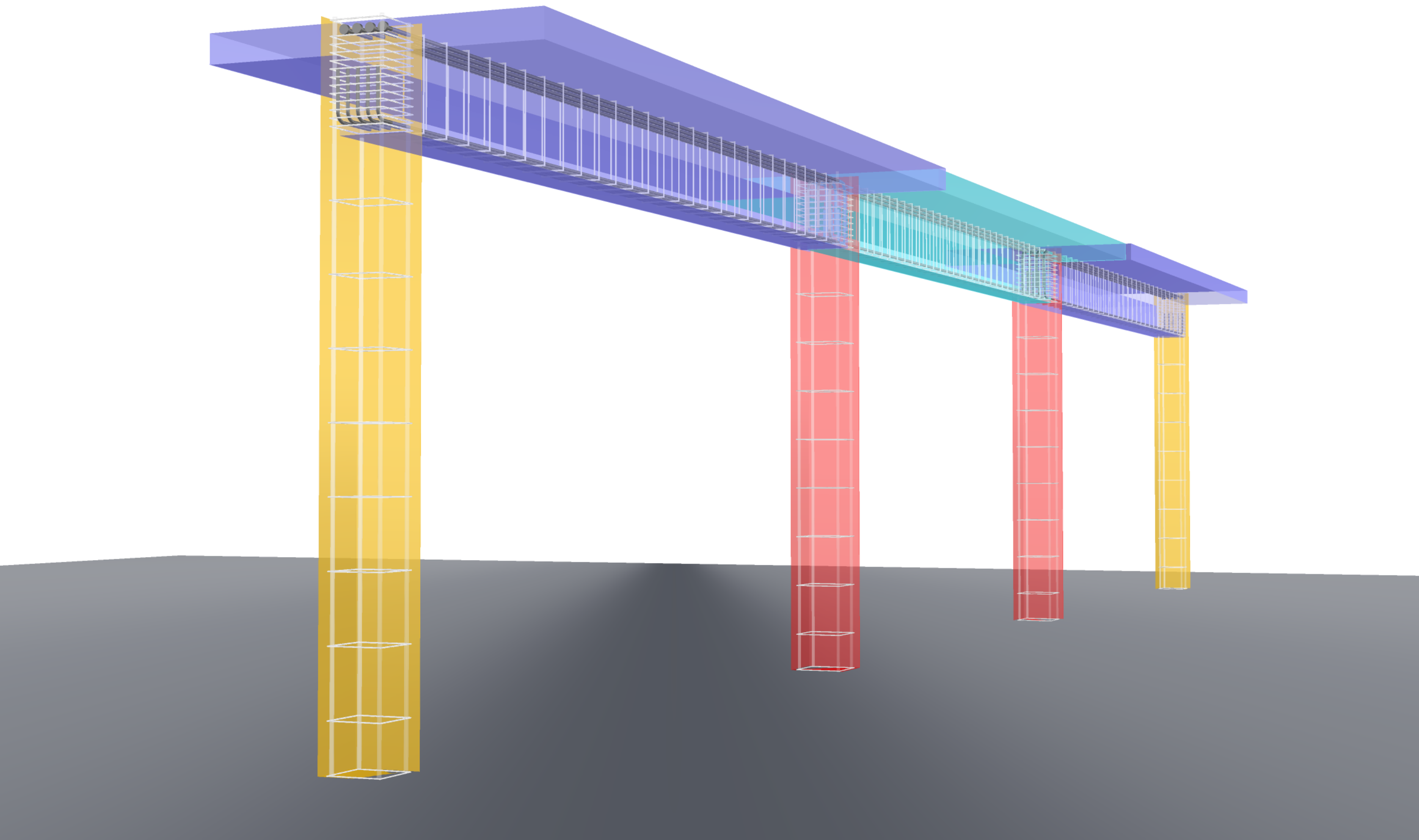

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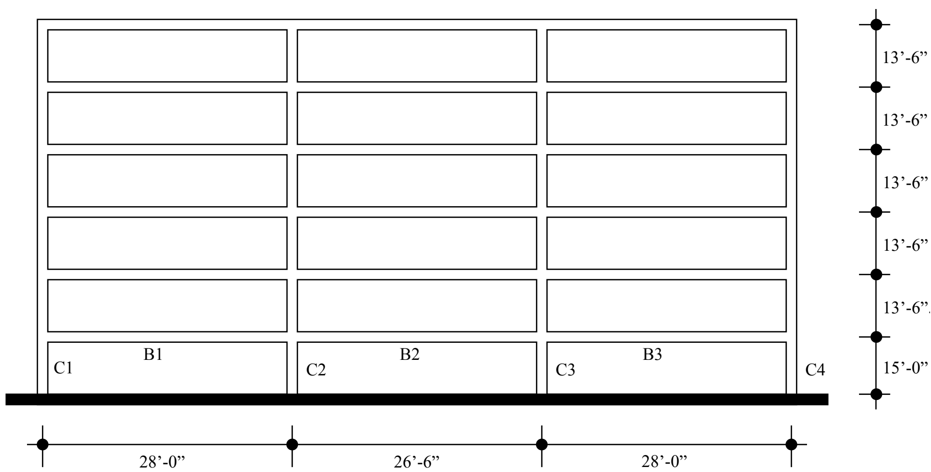
Executive Summary

Objective

Apply principles of ACI 318-19 to design the first floor beams and columns of the reinforced concrete frame shown below.

Project description

The frame section and dimensions are as follows, with the slab thickness, h = 8 in.



Provided Properties and Loading Conditions

The following project parameters are provided,

$f'_c = 6000$ psi $= 6$ ksi

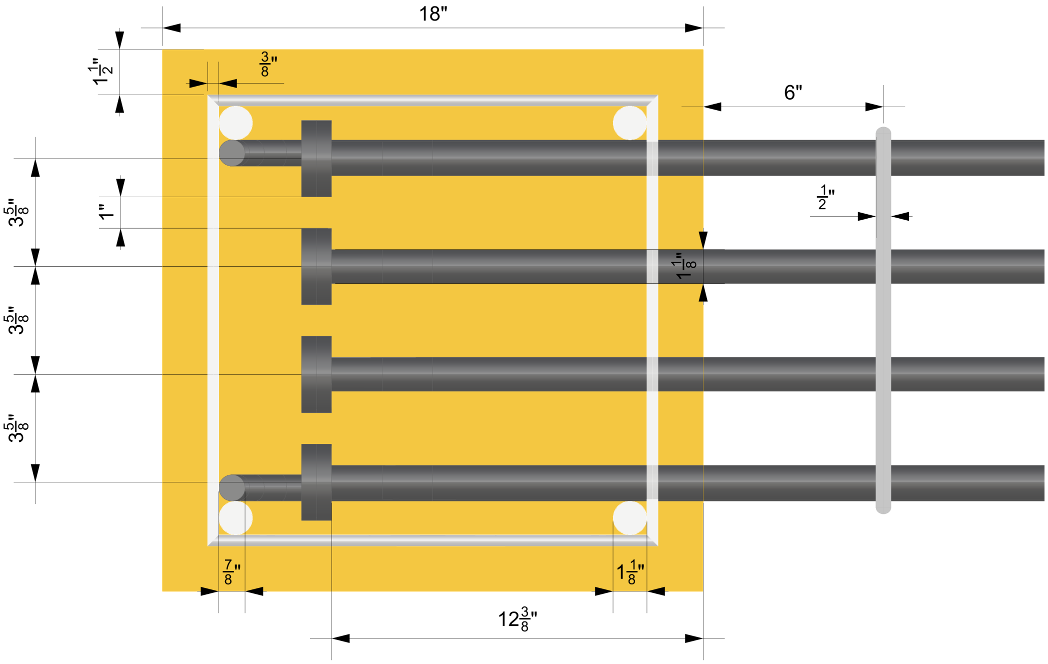
$f_y = 60$ ksi

$w_D = 1.6$ kips/ft

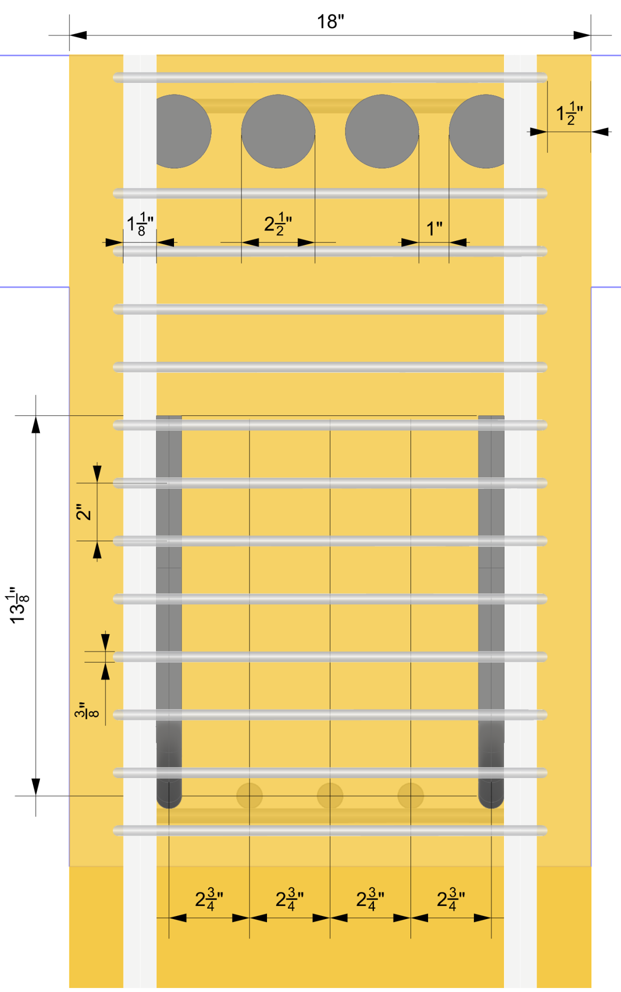
$w_L = 2.1$ kips/ft

Final Drawings

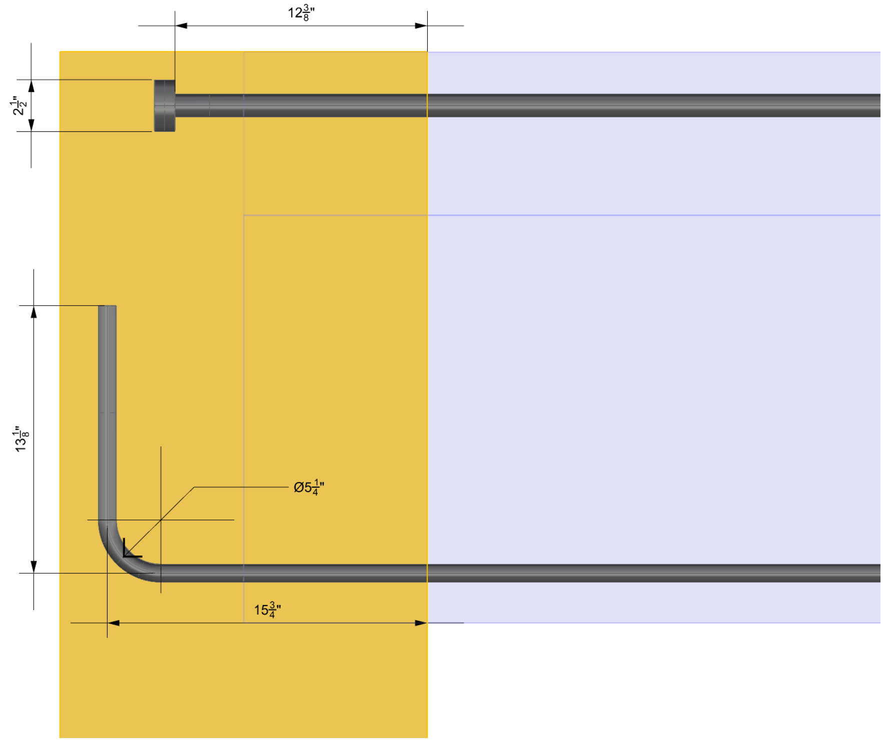
Exterior Column – Cross Section View



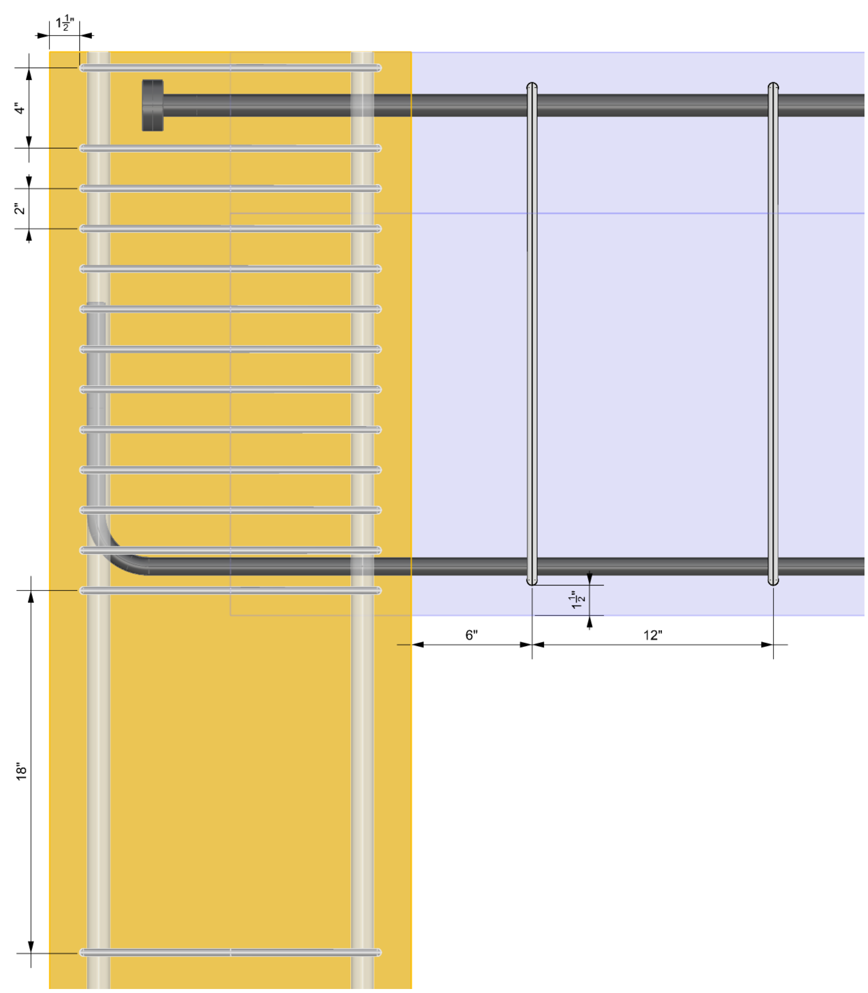
Exterior Column – Longitudinal Section View



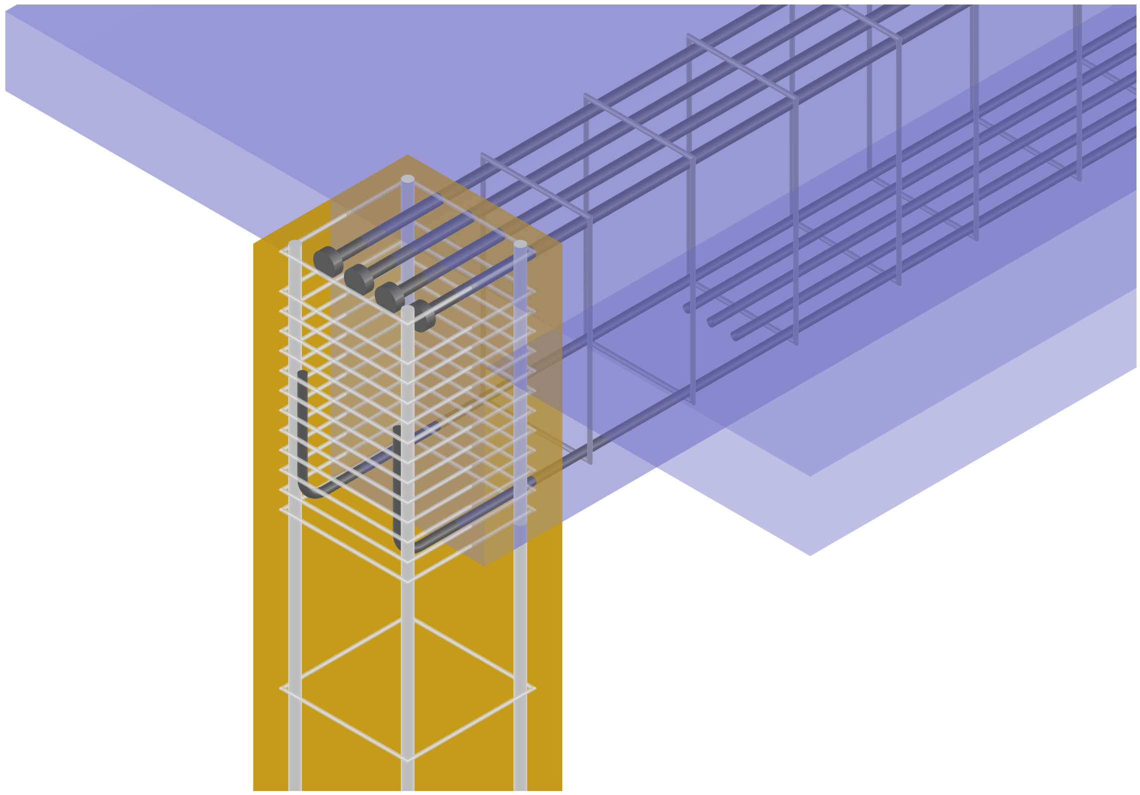
Development Length and Hook Diameter



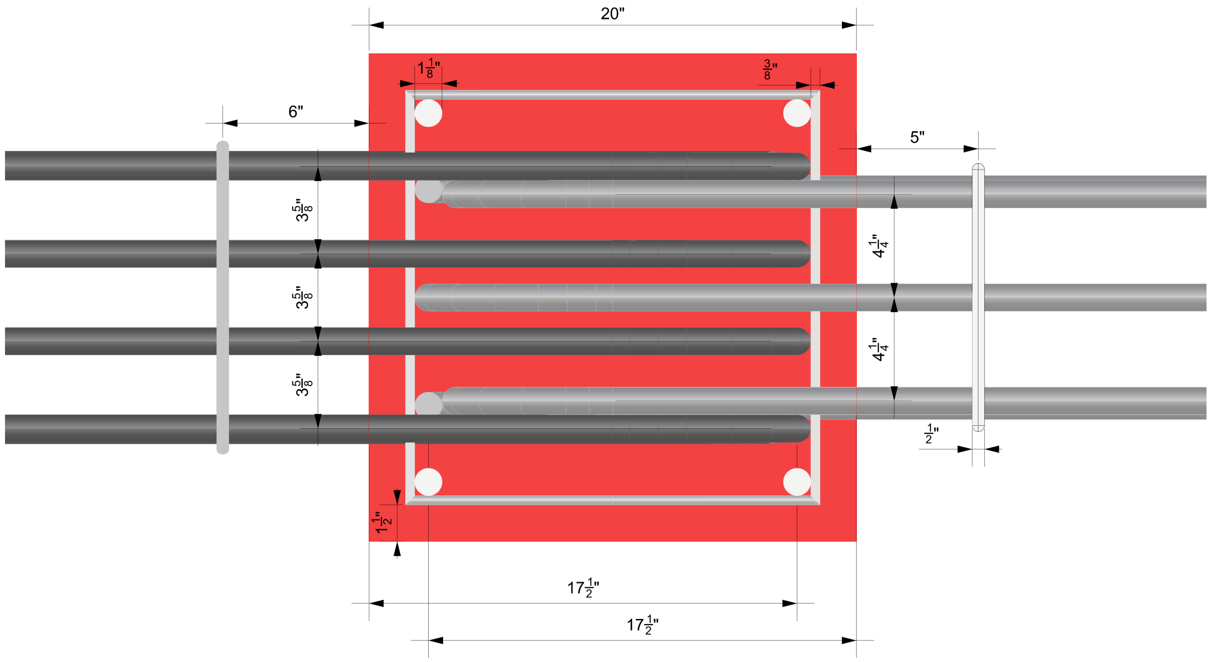
Exterior Column Tie and Stirrup Spacing



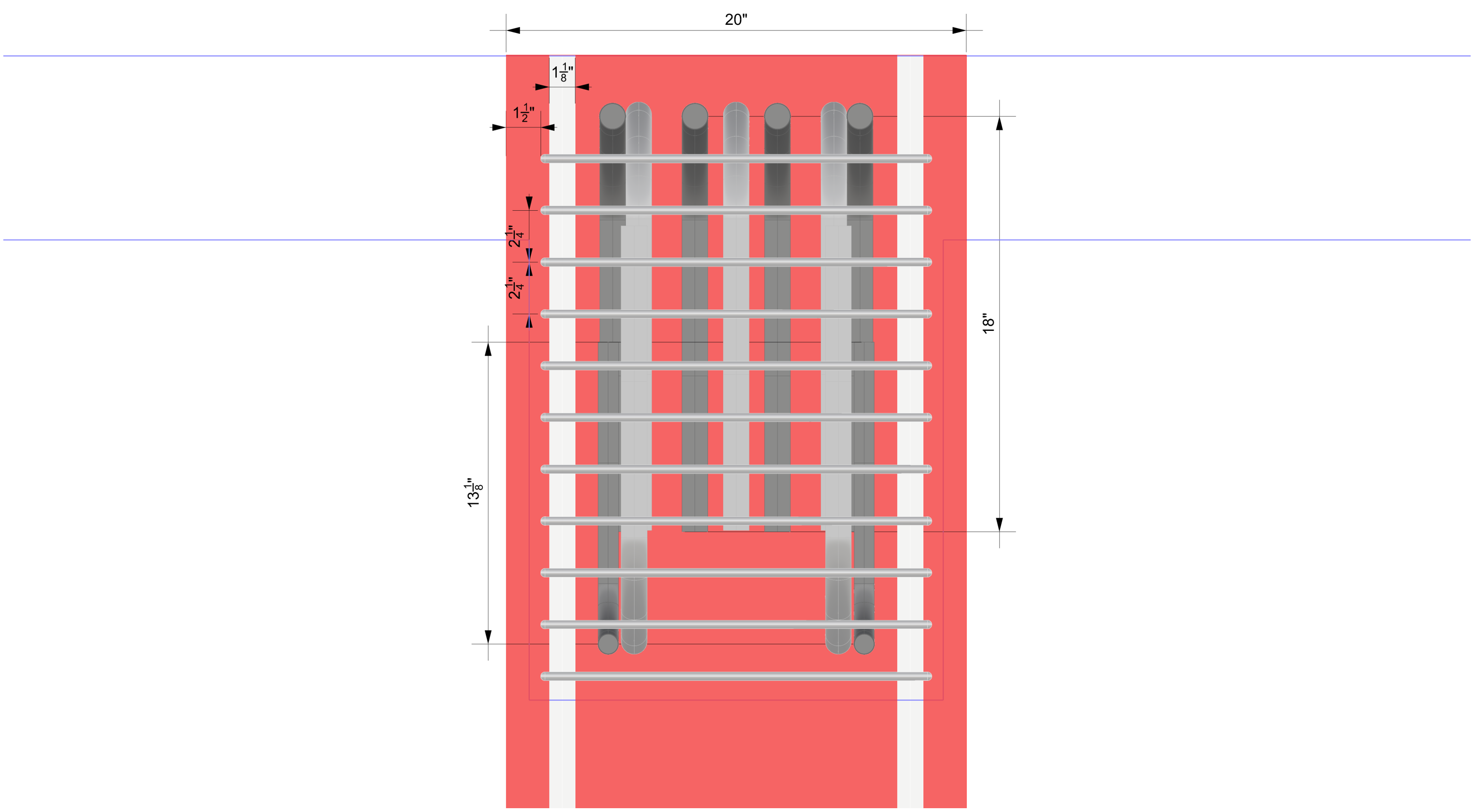
Exterior Column Isometric View



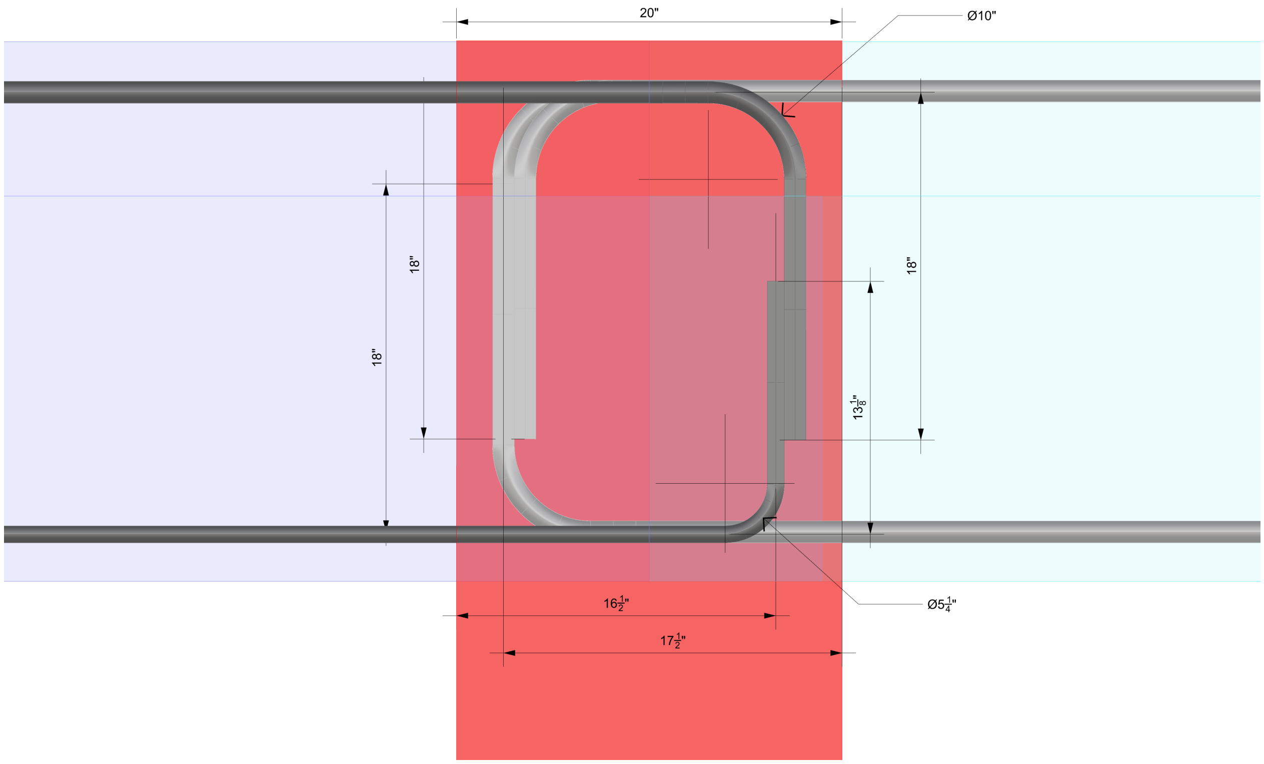
Interior Column – Cross Section View



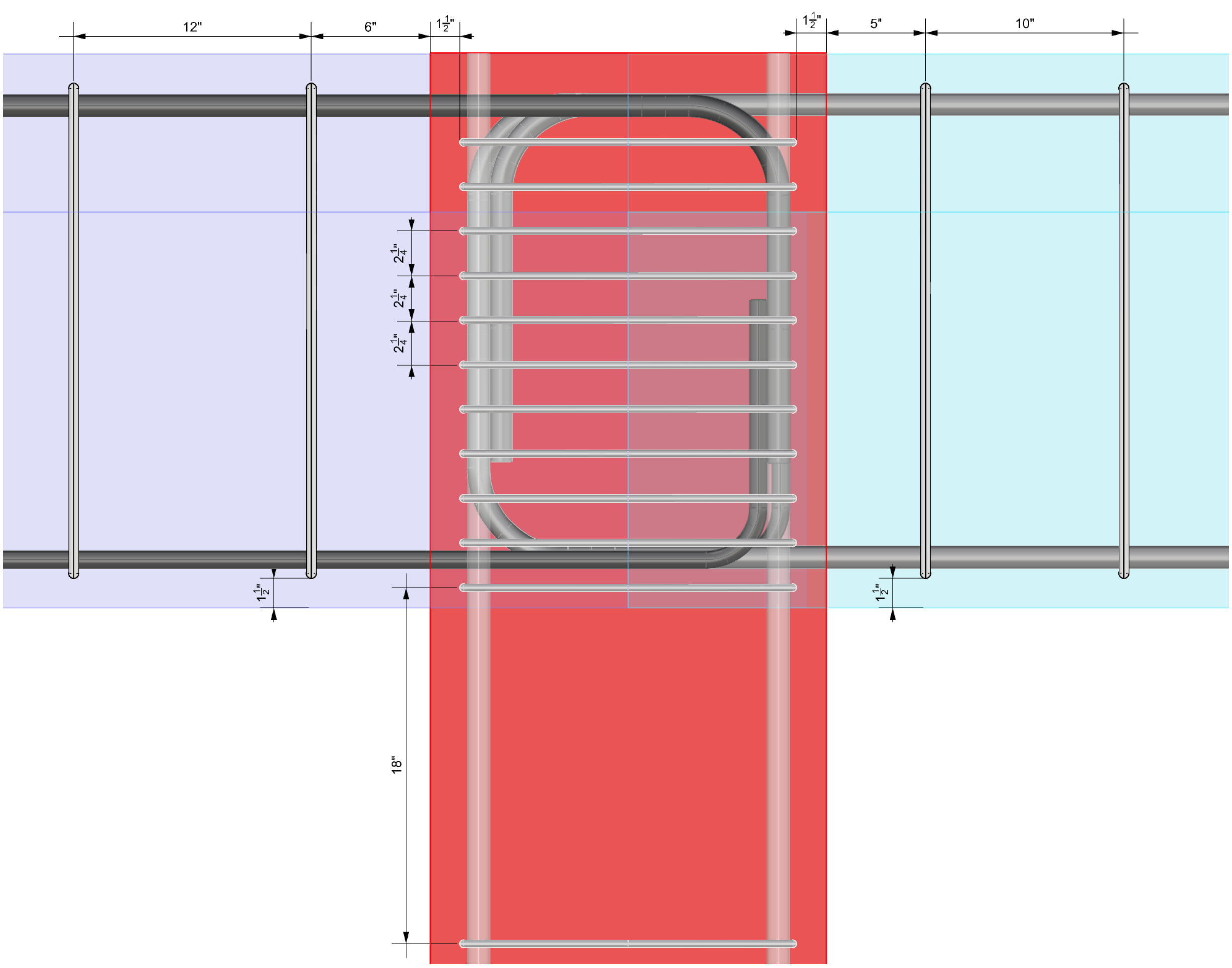
Interior Column – Longitudinal Section View



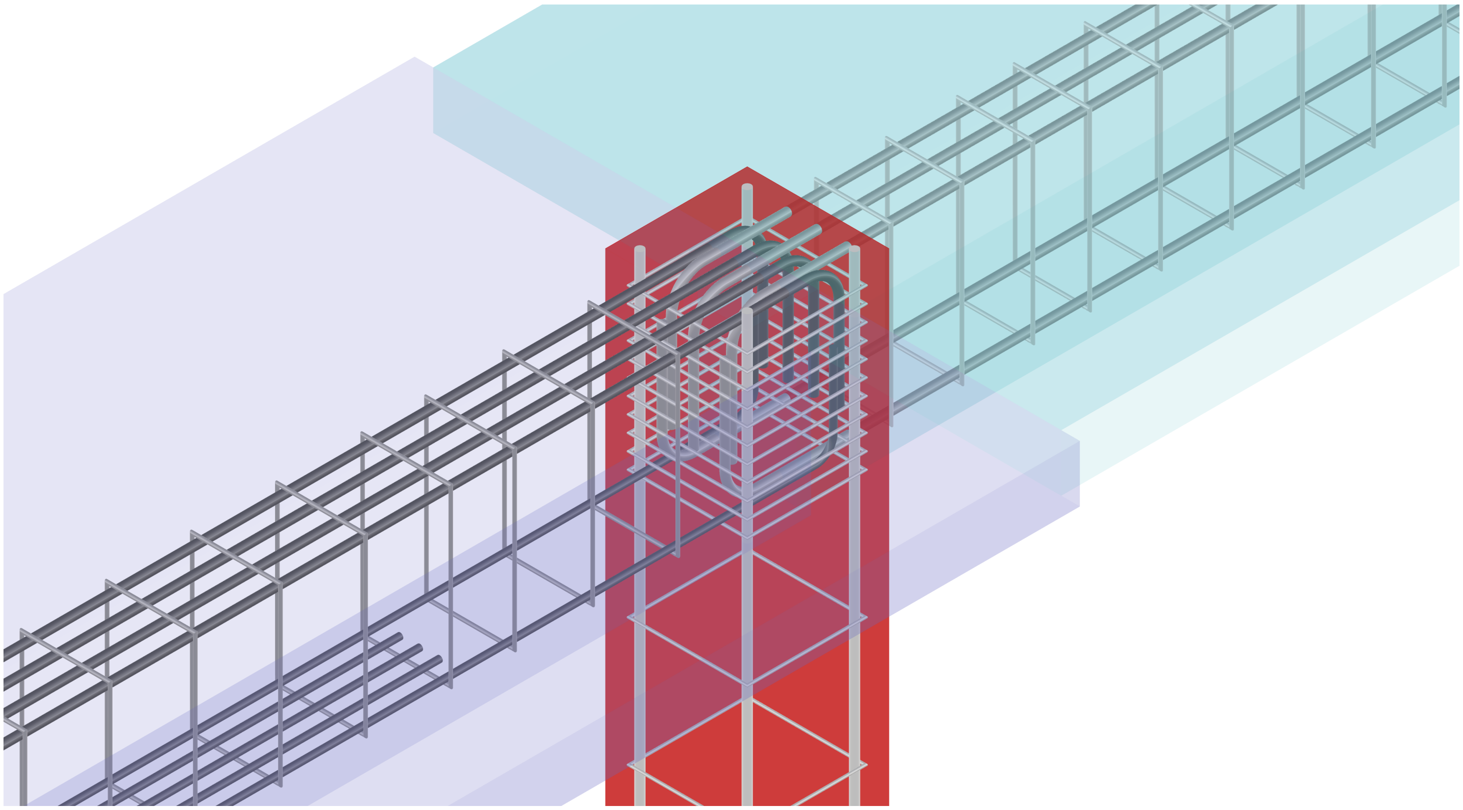
Development Length and Hook Diameter



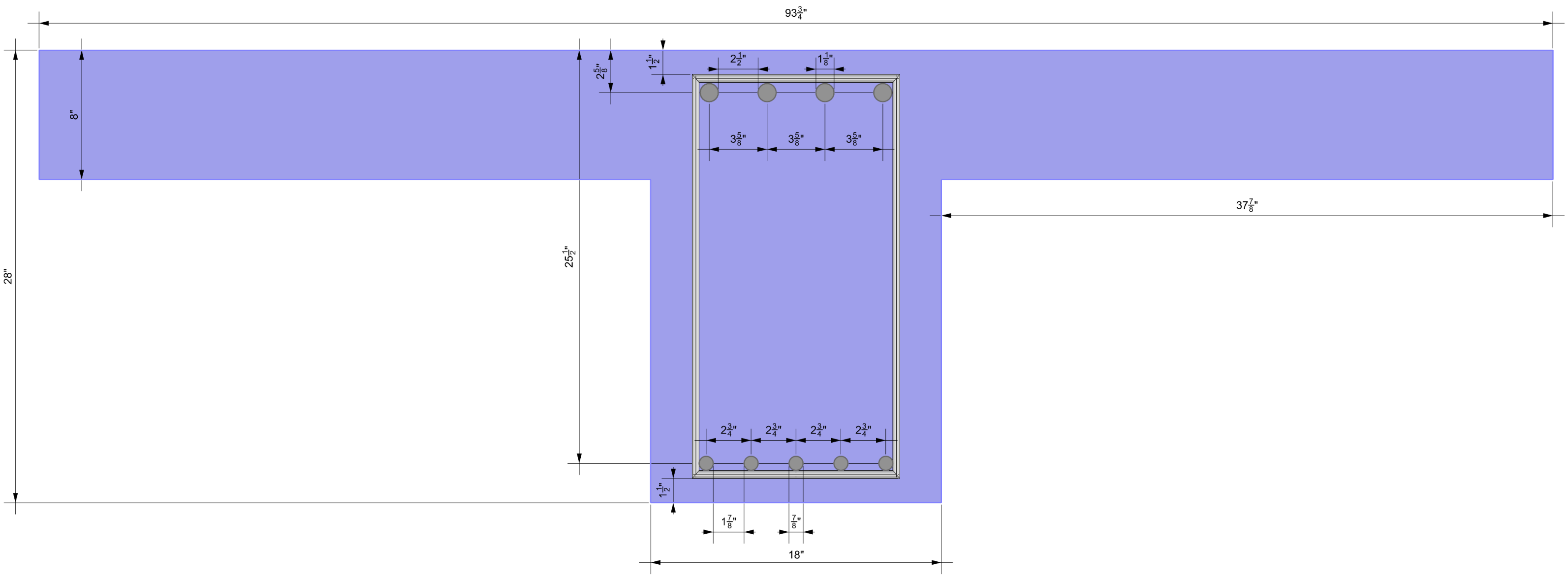
Interior Column Tie and Stirrup Spacing



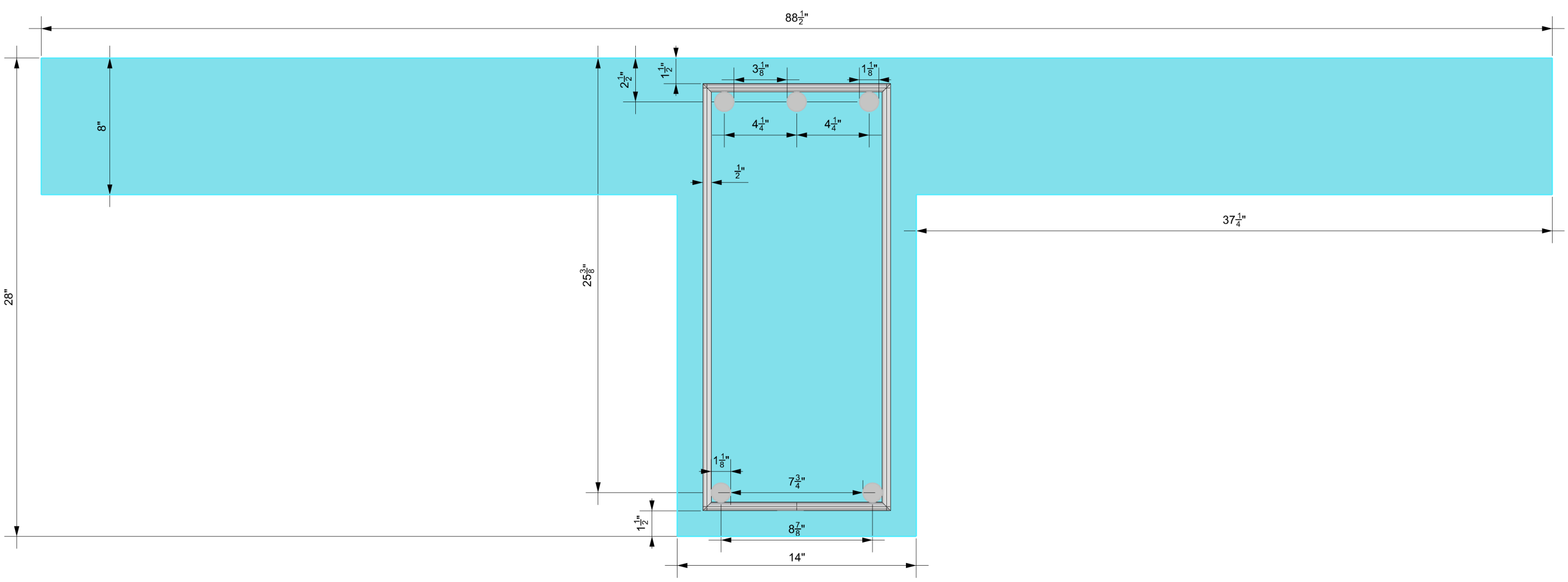
Interior Column Isometric View



Beam 1 / Beam 3 Section:



Beam 2 Section:



Appendix A: Design of Beams for Flexure

Load Calculation

Given:

$w_D = 1.6$ kips/ft

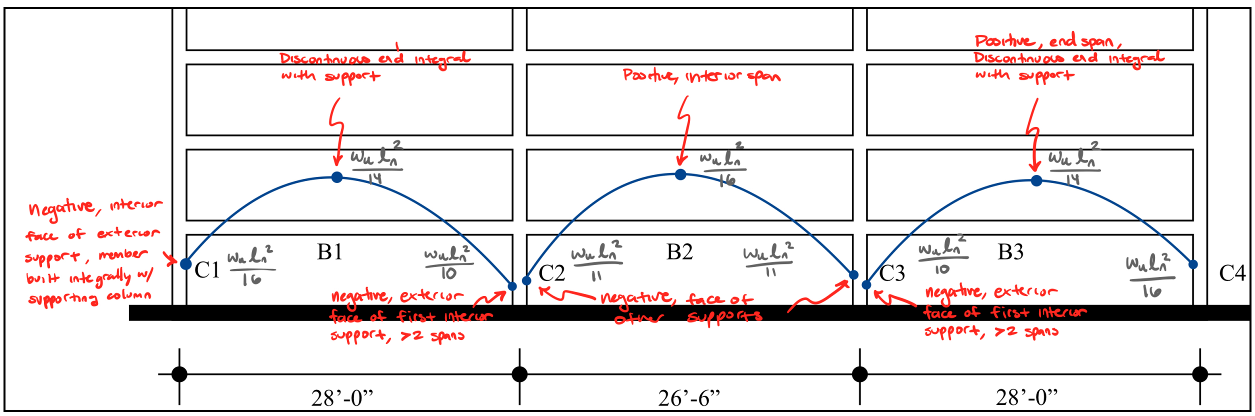
$w_L = 2.1$ kips/ft

Factored load, per PROVISION:

$$w_u = 1.2(w_D + w_{sw}) + 1.6(w_L) = 1.2(1.6) + 1.6(2.1) = 5.28 \text{ kips/ft}$$

Moment Calculations

Moment due to factored load, per Table 6.5.2, with updated clear-span calculations, such that:



$l_{n1} = 26.417$ ft

$l_{n1} = 24.833$ ft

$$M^+_{u,B1} = M^+_{u,B3} = \frac{w_u l_n^2}{14} = \frac{5.28 \text{ kips/ft } (26.417 \text{ ft})^2}{14} = 263.185 \text{ kips-ft}$$

$$M^-_{u,B1,max} = M^-_{u,B3,max} = \frac{w_u l_n^2}{10} = \frac{5.28 \text{ kips/ft } (26.417 \text{ ft})^2}{10} = 368.460 \text{ kips-ft}$$

$$M^+_{u,B2} = \frac{w_u l_n^2}{16} = \frac{5.28 \text{ kips/ft } (24.833 \text{ ft})^2}{16} = 203.509 \text{ kips-ft}$$

$$M^-_{u,B2} = \frac{w_u l_n^2}{11} = \frac{5.28 \text{ kips/ft } (24.833 \text{ ft})^2}{11} = 296.013 \text{ kips-ft}$$

Dimensional Approximation

Slab thickness, h = 8 in
Minimum height
Web width ratio
Effective flange width

> Section 6.3.2.1 of ACI 318-19 is used to approximate the flange width, *b*, where *h* is the slab thickness, s_w is the clear distance to the adjacent web, and l_n is the net span length.

Effective depth estimation
$d \approx h - 2.5$ in $= 25.5$ in

Moment arm estimation
Assume $J = 0.9$, $Jd = 0.9(25.5) = 22.95$ in $= 1.9125$ ft

Steel area required

$$\phi M_n = \phi A_s f_y Jd \ \Rightarrow A_{s,req} = \frac{M_u}{\phi f_y Jd}$$

Selection of reinforcing bars

Appendix B			n bar calculation			
bar size	diameter	area	2.55	1.97	3.57	2.87
	in	in2				
3	0.375	0.11	23.17	17.91	32.43	26.06
4	0.500	0.20	12.74	9.85	17.84	14.33
5	0.625	0.31	8.22	6.36	11.51	9.25
6	0.750	0.44	5.79	4.48	8.11	6.51
7	0.875	0.60	4.25	3.28	5.95	4.78
8	1.000	0.79	3.23	2.49	4.52	3.63
9	1.128	1.00	2.55	1.97	3.57	2.87
10	1.270	1.27	2.01	1.55	2.81	2.26
11	1.410	1.56	1.63	1.26	2.29	1.84
14	1.693	2.25	1.13	0.88	1.59	1.27

Design steel area
Stress block depth calculation
Neutral axis depth
Summarized below:

Moment Capacity

$f'_c = 6000$ psi $= 6$ ksi

$f_y = 60$ ksi

Check for tension-controlled failure

Area of steel, A_s	$A_s^+ = 3.00$ in^2 , $A_s^- = 4.00$ in^2	$A_s^+ = 2.00$ in^2 , $A_s^- = 3.00$ in^2
POSITIVE MOMENT **Calculation of stress block area, assuming** $a \leq h_f = 8$ **in**	$a = \dfrac{A_s f_y}{0.85 f'_c b_f} = \dfrac{3.00 \text{ in}^2 (60 \text{ ksi})}{0.85(6 \text{ ksi})(93.5 \text{ in})} = 0.363 \text{ in}$	$a = \dfrac{A_s f_y}{0.85 f'_c b_f} = \dfrac{2.00 \text{ in}^2 (60 \text{ ksi})}{0.85(6 \text{ ksi})(88.5 \text{ in})} = 0.266 \text{ in}$
NEGATIVE MOMENT **Calculation of stress block area, assuming** $a \leq h_w = h - 8$ **in** $= 20$ **in**	$a = \dfrac{A_s f_y}{0.85 f'_c b_w} = \dfrac{4.00 \text{ in}^2 (60 \text{ ksi})}{0.85(6 \text{ ksi})(18 \text{ in})} = 2.614 \text{ in}$	$a = \dfrac{A_s f_y}{0.85 f'_c b_f} = \dfrac{3.00 \text{ in}^2 (60 \text{ ksi})}{0.85(6 \text{ ksi})(14 \text{ in})} = 2.521 \text{ in}$
Rectangular compression zone?	Yes	Yes
$\beta_1 = 0.75$	$\beta_1 = 0.85 - \dfrac{0.05(f'_c - 4000)}{1000}$ if $4000 \text{ psi} < f'_c < 8000 \text{ psi}$	
POSITIVE $a = \beta_1 c$	$c = a/0.75 = 0.484 \text{ in}$	$c = a/0.75 = 0.354 \text{ in}$
NEGATIVE $a = \beta_1 c$	$c = a/0.75 = 3.486 \text{ in}$	$c = a/0.75 = 3.361 \text{ in}$

Check clear spacing with spacing minimums

Check moment capacity'=

Appendix B: Design of Beams for Shear

Shear Demand

Factored load

$w_u = 1.2(w_D + w_{sw}) + 1.6(w_L) = 1.2(1.6) + 1.6(2.1) = 5.28$ kips/ft

Factored live load

$w_{Lu} = 1.6(w_L) = 1.6(2.1) = 3.36$ kips/ft

Shear envelopes

Developed in excel, with references to the developing clear spans. The final calculations can be summarized as:

Beam 1 clear span	Beam 2 clear span	Beam 3 clear span
ft	ft	ft
26.417	24.833	26.417

	left end shear demand	mid span shear demand	right end shear demand	left end shear demand	mid span shear demand	right end shear demand	left end shear demand	mid span shear demand	right end shear demand
		kips			kips			kips	
Vu	69.74	11.095	80.201	65.56	10.43	65.56	80.201	11.095	69.74
Vn=Vu/phi	92.987	14.793	106.935	87.413	13.907	87.413	106.935	14.793	92.987

Shear Contributions

Concrete shear contribution

$V_c = 2\sqrt{f'_c}b_w d$, where $\phi = 0.75$ for shear (when $A_v \geq A_{v,min}$ and axial load = 0).

Strength reduction factor, ϕ

Stirrup area:

$A_v = 2(0.11 \text{ in}^2) = 0.22 \text{ in}^2$

Critical Section

critical section x=d=25.5in			critical section x=d=25.5in			critical section x=d=25.5in		
d		d	d		d	d		d
ft		ft	ft		ft	ft		ft
2.13		2.13	2.13		2.13	2.13		2.13
V_n(d)		V_n(d)	V_n(d)		V_n(d)	V_n(d)		V_n(d)
kips		kips	kips		kips	kips		kips
80.41		92.11	74.83		74.83	92.11		80.41

Max Spacing Calculations

Check section adequacy

	check section adequacy			check section adequacy			check section adequacy		
4*phi Vc	Vu,max	Vu-phi Vc	< 4*phi Vc	Vu,max	Vu-phi Vc	< 4*phi Vc	Vu,max	Vu-phi Vc	< 4*phi Vc
kips	kips	kips		kips	kips		kips	kips	
213.323923	80.201	26.870	true	65.56	24.080	true	69.74	16.409	true

Stirrup Spacing

Calculate stirrup spacing

Check for high shear

Governing spacing

Check for changes in spacing

B1/3 stirrup spacing, left	stirrup spacing	B1/3 stirrup spacing, right
in	in	in
12.89	10.48	15.38
12	10	15

low or high shear?			low or high shear?			low or high shear?		
Vn-Vc	? < ?	2Vc	Vn-Vc	? < ?	2Vc	Vn-Vc	? < ?	2Vc
35.83	low shear	142.22	32.11	low shear	142.22	21.88	low shear	142.22

max stirrup spacing			max stirrup spacing			max stirrup spacing		
d/2	24	governs?	d/2	24	governs?	d/2	24	governs?
12.75	24	12.75	12.75	24	12.75	12.75	24	12.75

max stirrup spacing			max stirrup spacing			max stirrup spacing		
		governs?			governs?			governs?
14.67	12.62	12.62	14.67	12.62	12.62	14.67	12.62	12.62

First stirrup from face of support, 8.7.6.3 , max d/2

Bar Cut-Offs

For beams 1 and 3, reinforcement may be terminated when the moment demand is less than $0.4M_u$. The reduction in area reflect this change by being 40% of the designed steel area. For beams 1 and 3, $0.4A_S = 0.4(3) = 1.2$ in^2

Since #5 bars are used, the three central bars may be terminated at approximately $0.17l_n$ from the outer-most support, as seen in the below design aids.

$0.17l_n = 0.17(26.417 \text{ ft}) \approx 4.5$ ft

Similarly, for the inner-most support,

$0.20l_n = 0.20(26.417 \text{ ft}) \approx 5.3$ ft

The above theoretical cutoff points are reduced per 9.7.3.3, which states reinforcing shall extend by the greater of:

(a) $d \approx 25.5$ in

(b) $12d_b = 12(0.875) = 10.5$ in

This, the actual cutoff points are:

Outer-most support: 4.5 ft $- 25.5$ in $= 2.3$ ft

Inner-most support: 5.3 ft $- 25.5$ in $= 3.1$ ft

Beam 2 is excluded from the above cutoff calculation as it already has the minimum bar amount needed to place stirrups along the beam.

A similar methodology follows for the cutoff points of reinforcement in the beam flanges: Per Fig. R9.7.3.3, top bars must extend past their theoretical cutoff point by the greater of (d, $12d_b$, $l_n/16$) past the point of inflection.

Again, referring to the design aid diagrams, the theoretical cutoff points for beams 1 and 3 are:

$0.164l_n = 0.164(26.417$ ft$) \approx 4.3$ ft

$0.24l_n = 0.24(26.417$ ft$) \approx 6.3$ ft

Actual cutoffs:

Outer-most suport: 4.3 ft $+ 25.5$ in $= 6.8$ ft

Inner-most suport: 6.3 ft $+ 25.5$ in $= 8.5$ ft

Due to the use of 4 #9 bars for beams 1 and 3, only beam 2 saw the above cutoffs applied.

Appendix C: Design of Beam Anchorage

Development Lengths

Per 25.4.2.1, and values in 25.4.2.5, development length shall be the greater of:

(a) $l_d = \left(\frac{f_y \, \psi_t \, \psi_e \, \psi_g}{20\lambda \sqrt{f_c'}} \right) d_b$

(b) 12 in

		Beam 1 /3		Beam 2	
		M+ (web)	**M- (flange)**	**M+ (web)**	**M- (flange)**
		#7	#9	#9	#9
		0.875	1.128	1.128	1.128
provisions		**development length l_d (in)**			
fy	60	33.89	43.69	43.69	43.69
psi_t	1				
psi_e	1				
psi_g	1				
lambda	1				
fc'	6000				

Given the column dimension constraints, hooked anchorage is needed.

Hook Development Lengths

Per 25.4.3, and values in 25.4.3.2, development length shall be the greater of:

(a) $l_{dh} = \left(\frac{f_y \, \psi_e \, \psi_r \, \psi_o \, \psi_c}{55\lambda \sqrt{f_c'}} \right) d_b^{1.5}$

(b) $8d_b$

(c) 6 in

		Beam 1 /3		Beam 2	
		M+ (web)	**M- (flange)**	**M+ (web)**	**M- (flange)**
		#7	#9	#9	#9
		0.875	1.128	1.128	1.128
provisions		**hook development length l_dh (in)**			
fy	60	11.53	16.87	16.87	16.87
psi_e	1				
psi_r	1				
psi_o	1				
psi_c	1				
lambda	1				
fc'	6000				

Given the column constraints of the exterior column, headed bars are considered for reinforcement in the flanges of beams 1 and 3.

Hook Straight Extension

Per 25.3.1, for a 90 degree hook in the bars of interest, #7 and #9, the extension length and bar diameter minimum are as follows:

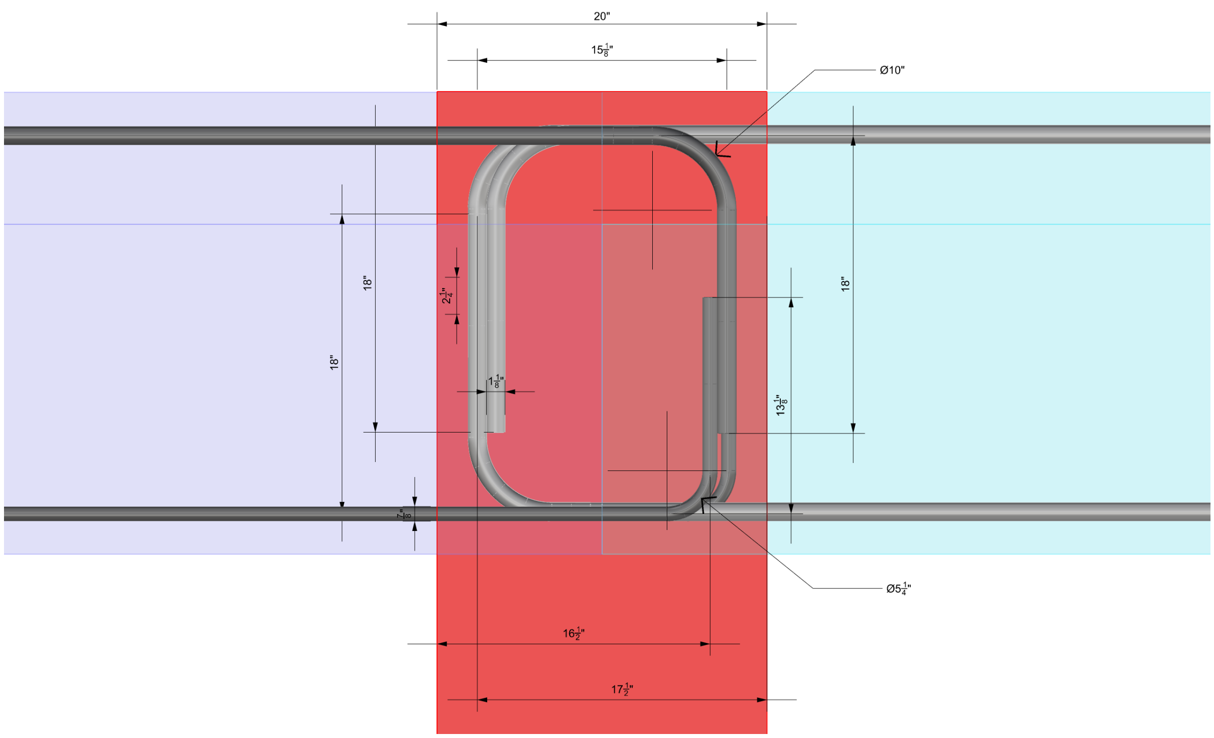
Straight extension, $l_{ext} = 12d_b$

Minimum inside diameter for #7 bar, $D = 6d_b$

Minimum inside diameter for #9 bar, $D = 8d_b$

	hook straight extension l_ext (in)			
90 degree	10.50	13.54	13.54	13.54
minimum bend diam.	6 d_b	8 d_b	8 d_b	8 d_b
D	5.25	9.024	9.024	9.024

These provisions are reflected in the final design, with proper anchorage in mind, the bars are developed until the straight extension is flush with column ties.



Dimensional Considerations

Some iterative design choices could have been avoided had anchorage been considered in tandem to the choice of reinforcement. In this process, the web width of beams 1 and 3 was widened from 14” to 18”, to account for problematic overlaps at the joint boundary of the interior beams. Hence, the consideration of headed bars.

Headed Bar Development Length

Per 25.4.4.2, the development length for headed bars shall be the longest of:

(a) $l_{dt} = \left(\dfrac{f_y \, \psi_e \, \psi_p \, \psi_o \, \psi_c}{75 \lambda \sqrt{f'_c}} \right) d_b^{1.5} = 12.37$

(b) $8d_b = 9.024$

(c) 6 in

Headed Bar Bearing Area

Per 25.4.4.1 (c), net bearing area of the head, A_{brg} shall be at least $4A_b$.

Since the headed bars in question are #9, $4A_b \geq 4$ in^2

Accounting for the bar area, the total head area should be greater than 5 in^2. It follows,

$$\pi \left(\frac{d_{head}}{2} \right)^2 \geq 5 \text{ in}^2$$

$$\Rightarrow d_{head} = 2.5 \text{ in}$$

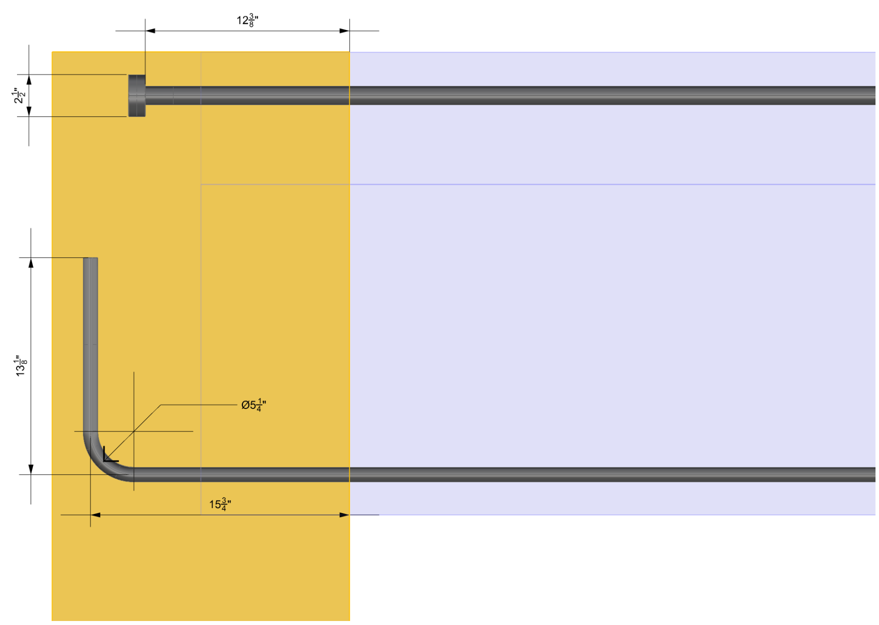
Dimensional Provisions

Per 25.4.4.1 (f), center-to-center facing between bars shall be at least $3d_b$,

$3d_b \geq 3(1.128 \text{ in}) = 3.38 \text{ in}$

With four bars, the column geometry allows for a center-to-center spacing of $3\frac{5}{8}$ in.

Development length, bearing area, and spacing provisions are reflected in the final design:



Appendix D: Design of Columns

Axial Loading

Approximate axial loading is calculated using (6.5.4):
For exterior columns:

$$P_{u1} = (n \text{ stories})\left(\frac{w_u l_{n1}}{2}\right)$$

$$P_{u1} = (6)(5.28 \text{ kips/ft})(28 \text{ ft})(1/2) = 443.52 \text{ kips}$$

For interior columns:

$$P_{u1} = (n \text{ stories})\left(\frac{1.15 w_u l_{n1}}{2} + \frac{w_u l_{n2}}{2}\right)$$

$$P_{u1} = (6)\left(\frac{5.28 \text{ kips/ft}}{2}\right)\left((1.15)28 \text{ ft} + 26.5 \text{ ft}\right) = 929.81 \text{ kips}$$

Estimating axial capacity and dimensions

$\phi P_n = 0.8\phi(0.85 f_c' A_g)$, where $\phi = 0.65$ (for compression)

$$\phi P_n \cong 0.4 f_c' A_g = 0.4 f_c' h^2$$

$$\phi P_n \geq P_u$$

$$0.4(6000 \text{ psi})(h_{ext}^2) \geq 443.52 \text{ kips}$$

$h \approx$ 14 in

$$0.4(6000 \text{ psi})(h_{int}^2) \geq 929.81 \text{ kips}$$

$h \approx$ 20 in

Dimension minimums and practical considerations
After the design of anchorage, 14 inches proved to be too small, this dimension is increased to 18” in the final design.

Steel Area Calculation

Gross area

$$A_g = 196 \text{ in}^2$$

$$A_g = 400 \text{ in}^2$$

Minimum steel area

$$A_{s,req} = A_g(0.01) = 1.96 \text{ in}^2$$

$$A_{s,req} = A_g(0.01) = 4.00 \text{ in}^2$$

Reinforcing Geometry

Reinforcing minimums of four longitudinal bars, as well as iterative P–M diagram tests, led to the selection of 2% steel for exterior columns and 1% for the interior columns

Selection of steel bars:

Appendix B			**n bar calculation**	
bar size	diameter	area	3.92	4
	in	in2		
3	0.375	0.11	35.6	36.4
4	0.500	0.20	19.6	20.0
5	0.625	0.31	12.6	12.9
6	0.750	0.44	8.9	9.1
7	0.875	0.60	6.5	6.7
8	1.000	0.79	5.0	5.1
9	1.128	1.00	3.9	4.0
10	1.270	1.27	3.1	3.1
11	1.410	1.56	2.5	2.6
14	1.693	2.25	1.7	1.8

Factored Moments

Distribution factor, DF

$$DF = \frac{(1/L_{LC})}{(1/L_{LC}) + (1/L_{Uc})} = 0.474$$

Factored moments

Delta M

$\Delta M = M^{-}_{u,B2} - M^{-}_{u,B2} =$ kips-ft

Check the design load and moment combination in the respective P–M Diagram

P – M Diagrams

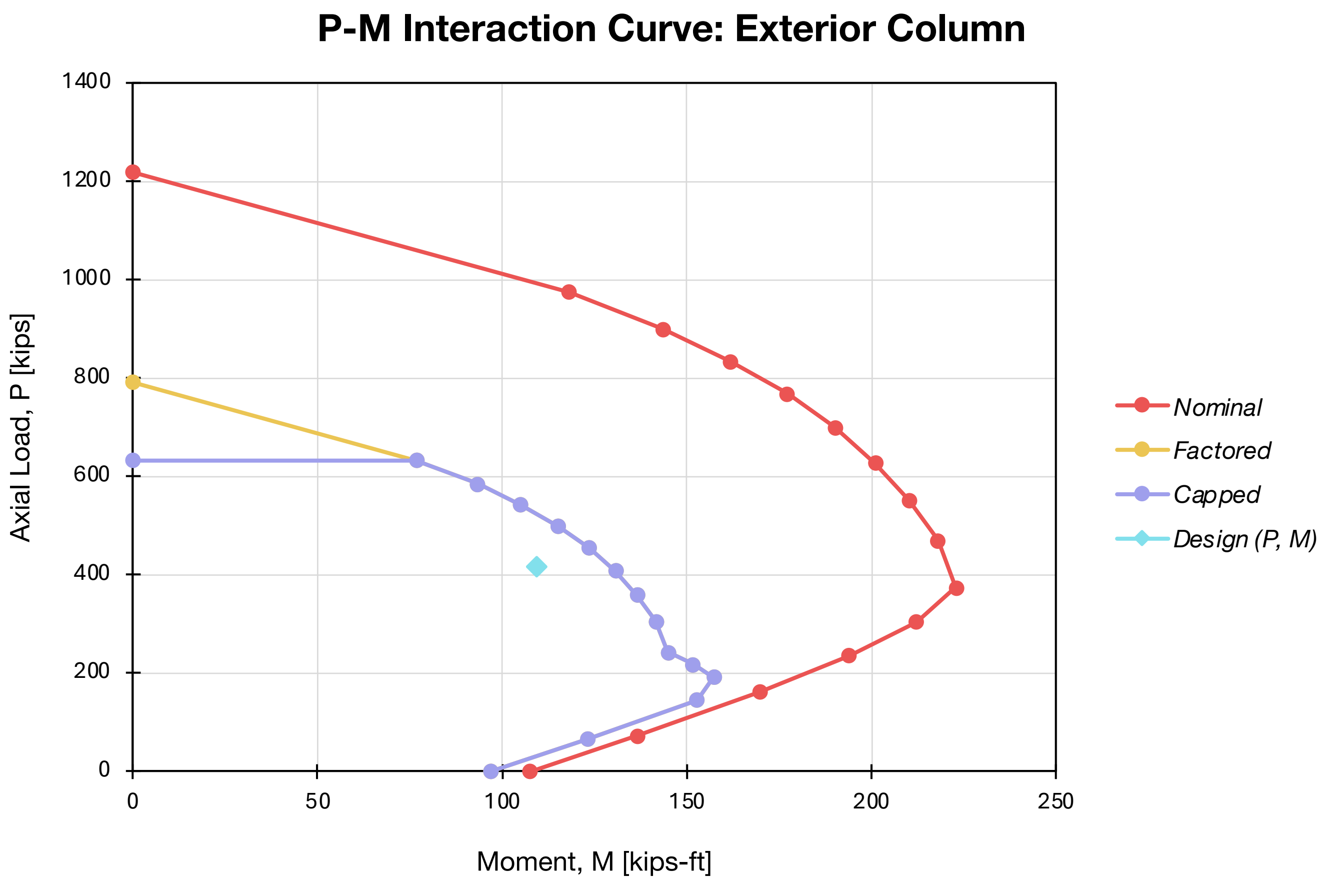
The following are evaluated using the clear spans of the beams, and the initial 14” and 20” column size approximations, prior to the dimensional corrections due to anchorage conflicts.

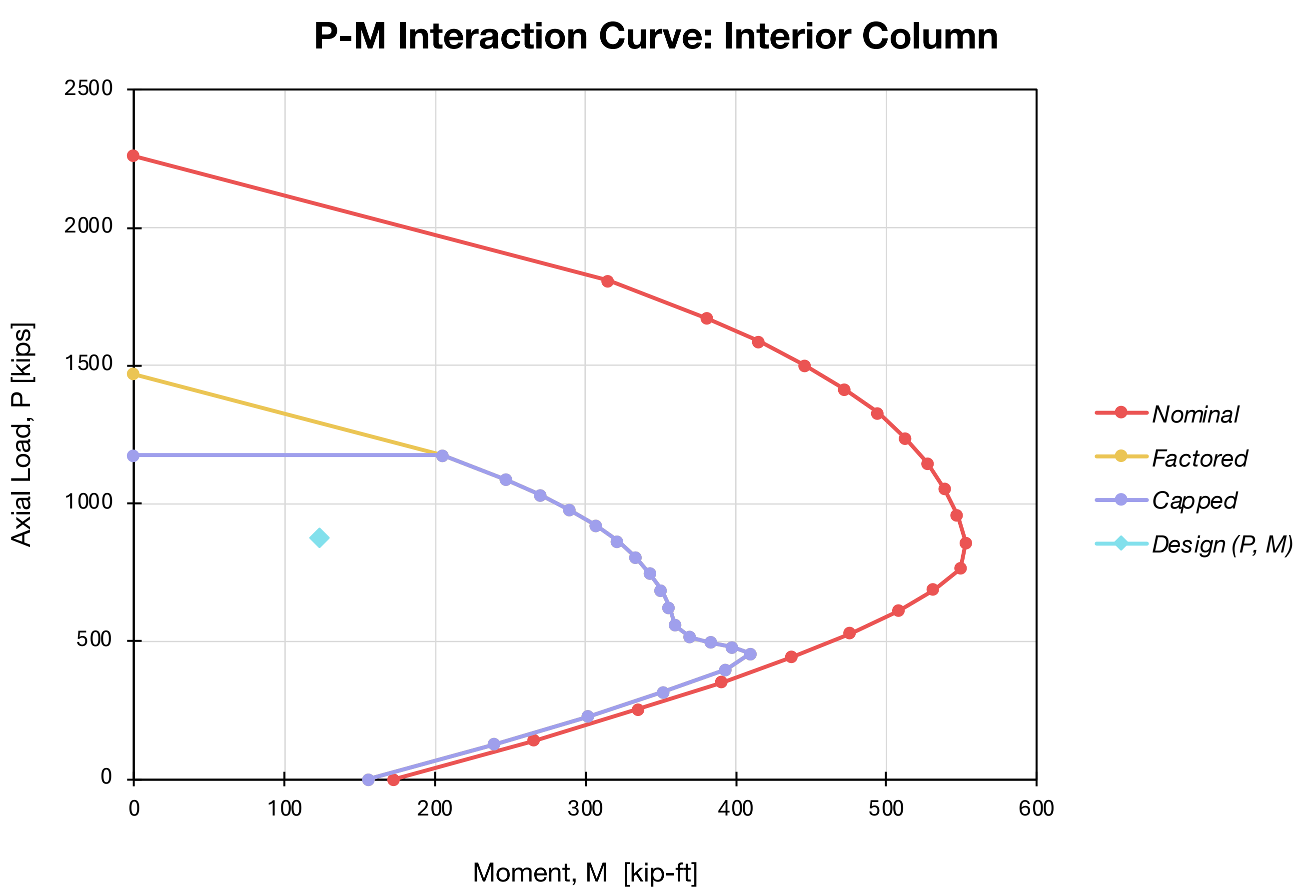
Factored nominal axial capacity

$$\phi P_{n,max} = \phi 0.80 P_o = \phi(0.80)\left[0.85 f'_c (A_g - A_{st}) + f_y A_{st}\right]$$

$\phi = 0.65$ for tied columns

Calculations further developed in excel.


P-M Interaction Curve: Exterior Column
Axial Load, P [kips]
Moment, M [kips-ft]
1400
1200
1000
800
600
400
200
0
0
50
100
150
200
250
Nominal
Factored
Capped
Design (P, M)


P-M Interaction Curve: Interior Column
Axial Load, P [kips]
Moment, M [kip-ft]
2500
2000
1500
1000
500
0
0
100
200
300
400
500
600
Nominal
Factored
Capped
Design (P, M)

Clear Spans

$$l_1 = 28 \text{ ft}$$

$$l_{n1} = l_1 - \frac{h_{c1}}{2} - \frac{h_{c2}}{2}$$

$$l_{n1} = 28 \text{ ft} - 9 \text{ in} - 10 \text{ in} = 26.417 \text{ ft}$$

$$l_2 = 26.5 \text{ ft}$$

$$l_{n2} = l_2 - \frac{h_{c2}}{2} - \frac{h_{c3}}{2}$$

$$l_{n1} = 26.5 \text{ ft} - 10 \text{ in} - 10 \text{ in} = 24.833 \text{ ft}$$